# Paraconsistency, information, and evidence

**Abilio Rodrigues Filho**

**Federal University of Minas Gerais, Brazil**

*abilio.rodrigues@gmail.com*

Universidade Federal Rural do Rio de Janeiro

Maio de 2019

# Overview

1. Paraconsistency as preservation of evidence
   - Conflicting evidence
   - Information *versus* evidence
2. Logics of evidence and truth
   - The Basic Logic of Evidence – *BLE*
   - The Logic of Evidence and Truth – $LET_J$
3. Semantics
   - Non-deterministic valuation semantics
   - Probabilistic semantics
   - Inferential semantics

# On paraconsistency

## What does it mean to accept a contradiction?

# Paraconsistent logics

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**What is the nature of contradictions that are accepted in paraconsistent logics?**

# Dialetheism: true contradictions

*A dialetheia is a sentence, A, such that both it and its negation, $\neg A$, are true (...) Dialetheism is the view that there are dialetheias. (...) dialetheism amounts to the claim that there are true contradictions.* (Priest and Berto, Dialetheism, Stanford.)

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*Such a contradictory object really exists??!!*
*I don't think so.*

# Contradictions as conflicting evidence

In order to:

1. reject dialetheism,
2. reject a metaphysically neutral position about contradictions, and
3. endorse a paraconsistent logic,

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- A property weaker that truth: a proposition $A$ may enjoy such a property even if $A$ is not true.
- 'Evidence that $A$ is true' $\rightsquigarrow$ 'reasons for believing in $A$',
- 'Evidence that $A$ is false' $\rightsquigarrow$ 'reasons for believing in $\neg A$'.
- Non-conclusive evidence can be *contradictory* and *incomplete*.

# Information vs. evidence

# Jon Michael Dunn on information

*I like to think of information, at least as a first approximation, as what is left from knowledge when you subtract, justification, truth, belief, and any other ingredients such as reliability that relate to justification. Information is, as it were, a mere "idle thought." Oh, one other thing, I want to subtract the thinker.*

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*[Information] is something like a Fregean "thought," i.e., the "content" of a belief that is equally shared by a doubt, a concern, a wish, etc.*

*(J. M. Dunn, Information in computer science, 2008, p. 589.)*

# Information versus evidence

- 'Bare-boned' information:
  1. a pure propositional content, expressible (in general) by language;
  2. objective;
  3. does not imply belief;
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- Non-conclusive evidence = bare-boned information
  + a degree of non-conclusive justification.
- Non-conclusive justification is a justification that might be wrong, that may end up not being a justification at all.
- Information is more general than evidence: evidence, even conclusive, is still information.

# The idea of a logic of evidence

# The idea of a basic logic of evidence

- Four scenarios with respect to the evidence for a proposition $A$:
  1. No evidence at all: both $A$ and $\neg A$ do not hold;
  2. Only evidence that $A$ is true: $A$ holds, $\neg A$ does not hold;
  3. Only evidence that $A$ is false: $A$ does not hold, $\neg A$ holds;
  4. Conflicting evidence: both $A$ and $\neg A$ hold.

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- A logic of evidence does not preserve truth, but rather evidence from premises to conclusion.
- Positive and negative evidence are two primitive, independent and non-complementary notions: absence of positive evidence $\neq$ existence of negative evidence, and so on.
- A logic of evidence has different rules for positive and negative evidence.

# The Basic Logic of Evidence – *BLE* (*N4*)

$$\frac{A \quad B}{A \wedge B}\ \wedge I \qquad \frac{A \wedge B}{A}\ \wedge E\ \frac{A \wedge B}{B} \qquad \frac{\begin{matrix}[A]\\ \vdots \\ B\end{matrix}}{A \to B}\ \to I \qquad \frac{A \to B \quad A}{B}\ \to E$$

$$\frac{A}{A \vee B}\ \vee I\ \frac{B}{A \vee B} \qquad \frac{A \vee B \quad \begin{matrix}[A]\\ \vdots \\ C\end{matrix} \quad \begin{matrix}[B]\\ \vdots \\ C\end{matrix}}{C}\ \vee E$$

$$\frac{\neg A \quad \neg B}{\neg(A \vee B)}\ \neg\vee I \qquad \frac{\neg(A \vee B)}{\neg A}\ \neg\vee E\ \frac{\neg(A \vee B)}{\neg B} \qquad \frac{A \quad \neg B}{\neg(A \to B)}\ \neg\to I \qquad \frac{\neg(A \to B)}{A}\ \neg\to E\ \frac{\neg(A \to B)}{\neg B}$$

$$\frac{\neg A}{\neg(A \wedge B)}\ \neg\wedge I\ \frac{\neg B}{\neg(A \wedge B)} \qquad \frac{\neg(A \wedge B) \quad \begin{matrix}[\neg A]\\ \vdots \\ C\end{matrix} \quad \begin{matrix}[\neg B]\\ \vdots \\ C\end{matrix}}{C}\ \neg\wedge E$$

$$\frac{A}{\neg\neg A}\ DNI \qquad \frac{\neg\neg A}{A}\ DNE$$

# Extending *BLE* to a logic of evidence and truth

# The logic of evidence and truth – $LET_J$

The Logic of Evidence and Truth ($LET_J$) is obtained by extending the language of $BLE$ with a classicality operator $\circ$ and adding the following inference rules:

$$\frac{\circ A \quad A \quad \neg A}{B}\ EXP^{\circ} \qquad\qquad \frac{\circ A}{A \vee \neg A}\ PEM^{\circ}$$

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- The operator $\circ$ works as a **context switch**: if $\circ A, \circ B, \circ C \ldots$ hold, the *argumentative context* of $A$, $B$, $C$... is classical.

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- The operator $\circ$ works as a **context switch**: if $\circ A, \circ B, \circ C \ldots$ hold, the *argumentative context* of $A, B, C\ldots$ is classical.
- A proposition $\circ A$ may be read as:
  1. $A$ behaves classically,
  2. conclusive evidence for $A$ or $\neg A$,
  3. reliable information about $A$ or $\neg A$,
  4. the truth-value of $A$ has been established.

# The intended interpretation of $LET_J$

- When $\circ A$ does not hold, four *non-conclusive* scenarios:
  1. $A$ holds, $\neg A$ doesn't $\rightsquigarrow$ only evidence that $A$ is true.
  2. $\neg A$ holds, $A$ doesn't $\rightsquigarrow$ only evidence that $A$ is false.
  3. Both $A$ and $\neg A$ don't hold $\rightsquigarrow$ no evidence at all.
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  3. Both $A$ and $\neg A$ don't hold $\rightsquigarrow$ no evidence at all.
  4. both $A$ and $\neg A$ hold $\rightsquigarrow$ conflicting evidence.
- When $\circ A$ holds, two *conclusive* scenarios:
  5. $A$ holds $\rightsquigarrow$ conclusive evidence that $A$ is true.
  6. $\neg A$ holds $\rightsquigarrow$ conclusive evidence that $A$ is false.

# Semantics

# Semantics for logics of evidence and truth

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# Semantics for logics of evidence and truth

- Non-deterministic **valuation semantics**: 'mathematical representations' of the deductive systems, rather than explanations of meanings.
- **Probabilistic semantics**: intends to quantify the evidence attributed to a proposition $A$ (joint work with J. Bueno-Soler and W. Carnielli).
- **Inferential semantics**: meanings are explained compositionally by means of the introductions rules, analogously to the proof-theoretic semantics for intuitionistic logic.

# Non-deterministic valuation semantics

- Given a language $L$, valuations are functions from the set of formulas of $L$ to $\{0, 1\}$ according to certain conditions that somehow 'represent' the axioms and/or rules of inference.
- The attribution of the value *0* to a formula $A$ means that $A$ *does not hold*, and the value *1* means that $A$ *holds*.

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- The attribution of the value *0* to a formula $A$ means that $A$ *does not hold*, and the value *1* means that $A$ *holds*.
- The valuation semantics for $BLE$ and $LET_J$:

1. Provide decision procedures by means of the so-called *quasi-matrices*
2. Are non-deterministic – the semantic value of $\neg A$ is not functionally determined by the semantic value of $A$.

Carnielli and Rodrigues. An epistemic approach to paraconsistency: a logic of evidence and truth. In *Synthese*, 2017. Preprint: http://bit.ly/SYNLETJ.

# Valuation semantics for $BLE$ and $LET_J$

$p \to (\neg p \to q)$

| $p$ | 0 | | | | | | | 1 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\neg p$ | 0 | | | | 1 | | | 0 | | | 1 | |
| $q$ | 0 | | | 1 | 0 | | 1 | 0 | | 1 | 0 | 1 |
| $\neg p \to q$ | 0 | | 1 | 1 | 0 | | 1 | 0 | 1 | 1 | 0 | 1 |
| $p \to (\neg p \to q)$ | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $s_5$ | $s_6$ | $s_7$ | $s_8$ | $s_9$ | $s_{10}$ | $s_{11}$ | $s_{12}$ |

# Decision procedure for $BLE$ and $LET_J$

$p \to (\neg p \to q)$ is invalid in $BLE$.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p$ | 0 | | | | | | | **1** | | | | |
| $\neg p$ | 0 | | | | **1** | | | 0 | | | **1** | |
| $q$ | 0 | | | 1 | **0** | | 1 | 0 | | 1 | **0** | 1 |
| $\neg p \to q$ | 0 | | 1 | 1 | 0 | | 1 | 0 | 1 | 1 | 0 | 1 |
| $p \to (\neg p \to q)$ | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $s_5$ | $s_6$ | $s_7$ | $s_8$ | $s_9$ | $s_{10}$ | $s_{11}$ | $s_{12}$ |

Given $s_{11}$, $s_1$ is a valuation.

# Decision procedure for $BLE$ and $LET_J$

$\circ p \to (p \vee \neg p)$ is valid in $LET_J$.

<table>
<tr><td>$p$</td><td colspan="4">0</td><td colspan="3">1</td></tr>
<tr><td>$\neg p$</td><td colspan="2">0</td><td colspan="2">1</td><td colspan="2">0</td><td>1</td></tr>
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</table>

$s_1$ is not a valuation because there is no $s'$ such that $s'(\circ p) = 1$ and $s'(p \vee \neg p) = 0$

# Probabilistic semantics

- Up to now, evidence was treated from a purely qualitative point of view.
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- In the classical approach to probabilities, $P(A) + P(\neg A) = 1$.
- Incomplete scenarios: little or no evidence for and against $A$.
  $P(A) + P(\neg A) < 1$
- Contradictory scenarios: conflicting evidence for $A$.
  $P(A) + P(\neg A) > 1$

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- $P(\circ A)$ may be read as expressing the degree of reliability of the evidence available for $P(A)$.
- $P(A)$ may be read as a measure of the risk associated to $A$, and $P(\circ A)$ may be the risk of the risk associated to $A$.
- $P(A)$ may even express the degree of belief in $A$, and $P(\circ A)$ the degree of reliability of this belief.

# From $LET_J$ to $LET_F$

Problems:

i. There is no plausible interpretation for the half-intuitionistic implication of $LET_J$ in probabilistic terms.

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Solutions:

i. We dropped the implication of *BLE*/*N4*. The result is the well-known Belnap-Dunn's logic of first-degree entailment (*FDE*).
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The logic so obtained we called the logic of evidence and truth based on $FDE$ – $LET_F$.

# The logic of first-degree entailment (*FDE*)

$$\frac{A \quad B}{A \wedge B} \wedge I \qquad \frac{A \wedge B}{A} \wedge E \ \frac{A \wedge B}{B}$$

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$$\frac{\neg A}{\neg(A \wedge B)} \neg \wedge I \ \frac{\neg B}{\neg(A \wedge B)} \qquad \frac{\neg(A \wedge B) \quad \begin{matrix}[\neg A] \\ \vdots \\ C\end{matrix} \quad \begin{matrix}[\neg B] \\ \vdots \\ C\end{matrix}}{C} \neg \wedge E$$

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$$\frac{A}{\neg\neg A} DN \ \frac{\neg\neg A}{A}$$

# Extending *FDE*: the logic $LET_F$

$LET_F = FDE +$ the following rules for $\circ$ and $\bullet$:

$$\frac{\circ A \quad A \quad \neg A}{B}\ EXP^{\circ} \qquad \frac{\circ A}{A \vee \neg A}\ PEM^{\circ}$$

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Derived rules: $\frac{A \quad \neg A}{\bullet A}\ R1 \qquad \frac{}{\bullet A \vee A \vee \neg A}\ R2$

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$\circ A$ implies conclusive evidence (or reliable information).

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$$\frac{\circ A \quad A \quad \neg A}{B}\ EXP^{\circ} \qquad \frac{\circ A}{A \vee \neg A}\ PEM^{\circ}$$

$$\frac{\circ A \quad \bullet A}{B}\ Cons \qquad \frac{}{\circ A \vee \bullet A}\ Comp$$

Derived rules: $\frac{A \quad \neg A}{\bullet A}\ R1 \qquad \frac{}{\bullet A \vee A \vee \neg A}\ R2$

Either *there is* conclusive evidence, or *there is not* conclusive evidence.

$\circ A$ implies conclusive evidence (or reliable information).

Non-conclusive evidence (or unreliable information) implies $\bullet A$.

# Non-deterministic valuation semantics for $LET_F$

The quasi-matrix below displays the behavior of $\circ$ and $\bullet$ in $LET_F$.

| $A$ | 0 | | | 1 | | |
|---|---|---|---|---|---|---|
| $\neg A$ | 0 | 1 | | 0 | | 1 |
| $\circ A$ | 0 | 1 | 0 | 1 | 0 | 0 |
| $\bullet A$ | 1 | 0 | 1 | 0 | 1 | 1 |

- Conflicting evidence implies $v(\bullet A) = 1$ and $v(\circ A) = 0$.
- No evidence at all implies $v(\bullet A) = 1$ and $v(\circ A) = 0$.
- If exactly one holds between $A$ and $\neg A$, then $v(\bullet A)$ and $v(\circ A)$ are undetermined.

In order to say that $A$ is true, or false, evidence for the truth, or for the falsity, of $A$ is not enough. We need conclusive evidence.

# The intended interpretation of $LET_F$

- When $\bullet A$ holds, four *non-conclusive* scenarios:
  1. $A$ holds, $\neg A$ doesn't $\rightsquigarrow$ only evidence that $A$ is true.
  2. $\neg A$ holds, $A$ doesn't $\rightsquigarrow$ only evidence that $A$ is false.
  3. Both $A$ and $\neg A$ don't hold $\rightsquigarrow$ no evidence at all.
  4. both $A$ and $\neg A$ hold $\rightsquigarrow$ conflicting evidence.

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  3. Both $A$ and $\neg A$ don't hold $\rightsquigarrow$ no evidence at all.
  4. both $A$ and $\neg A$ hold $\rightsquigarrow$ conflicting evidence.
- When $\circ A$ holds, two *conclusive* scenarios:
  5. $A$ holds $\rightsquigarrow$ conclusive evidence that $A$ is true.
  6. $\neg A$ holds $\rightsquigarrow$ conclusive evidence that $A$ is false.

# Back to probabilistic semantics

# Probability distributions

Given a logic $\mathcal{L}$, with a derivability relation $\vdash$ and a language $L$, a probability distribution for $\mathcal{L}$ is a real-valued function $P : L \mapsto \mathbb{R}$ satisfying the following conditions:

1. Non-negativity: $0 \leq P(A) \leq 1$ for all $A \in L$;
2. Tautologicity: If $\vdash A$, then $P(A) = 1$;
3. Anti-Tautologicity: If $A \vdash$, then $P(A) = 0$;
4. Comparison: If $A \vdash B$, then $P(A) \leq P(B)$;
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- These clauses define probability functions for an appropriate logic $\mathcal{L}$, just by taking $\vdash$ as the derivability relation of $\mathcal{L}$.
- These clauses define probability functions for both *FDE* and $LET_F$

# Completeness of the probabilistic semantics

**Definition**

A probabilistic semantic relation $\Vdash_P$ for $LET_F$ is defined as: $\Gamma \Vdash_P A$ if and only if for every probability function $P$, if $P(B) \geq \lambda$ for every $B \in \Gamma$, then $P(A) \geq \lambda$.

**Theorem**

Completeness of $LET_F$ with respect to probabilistic semantics:
$\Gamma \vdash A$ if and only if $\Gamma \Vdash_P A$

Bueno-Soler, J. W. Carnielli, and A. Rodrigues. Measuring evidence: a probabilistic approach to an extension of Belnap-Dunns Logic. Manuscript in preparation.

# What is next?

# The idea of an 'information space'

- The probabilistic semantics is not really talking about events, but rather about the information related to such events, constituted by propositions $A, \neg A, \bullet A, \circ A$, and other propositions formed with them.
- These propositions represent evidence that can be non-conclusive, incomplete, contradictory, more reliable or less reliable, and sometimes conclusive.

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- These propositions represent evidence that can be non-conclusive, incomplete, contradictory, more reliable or less reliable, and sometimes conclusive.
- **These propositions together with the measures of probabilities attributed to them by a $LET_F$-probability distribution we call an information space.**
- An information space is divided by $LET_F$ in parts that are exhaustive but may be non-exclusive.
- We cannot rely on the classical, mutually exclusive partitions of the sample space.

# Propagation of classicality

Propagation rules:

$$\frac{\circ A}{\circ\circ A} \qquad \frac{\circ\circ A}{\circ A} \qquad \frac{\circ A}{\circ\neg A} \qquad \frac{\circ\neg A}{\circ A}$$

Introduction rules $\wedge$:

$$\frac{\circ A \quad \neg A}{\circ(A \wedge B)} \qquad \frac{\circ B \quad \neg B}{\circ(A \wedge B)} \qquad \frac{\circ A \quad \circ B}{\circ(A \wedge B)}$$

Introduction rules $\vee$:

$$\frac{\circ A \quad A}{\circ(A \vee B)} \qquad \frac{\circ B \quad B}{\circ(A \vee B)} \qquad \frac{\circ A \quad \circ B}{\circ(A \vee B)}$$

Elimination rules:

$$\frac{\circ(A \wedge B)}{\circ A \vee \circ B} \qquad \frac{\circ(A \vee B)}{\circ A \vee \circ B}$$

Joint work with Walter Carnielli and Andreas Kapsner

# Introduction of classicality

$$\frac{\circ A \quad A}{\circ(A \vee B)} \qquad \frac{\circ B \quad B}{\circ(A \vee B)}$$

If $A$ is true, $(A \vee B)$ is true, and so is classical (*m.m.* for B).
If $A$ is true, it cannot be that $A \vee B$ is false – it would imply $\neg A$ and triviality.

$$\frac{\circ A \quad \circ B \quad \neg A \quad \neg B}{\circ(A \vee B)}$$

If both $A$ and $B$ are false, $(A \vee B)$ is false, and so classical (*m.m.* for B).

# Derivable rules for • (non-classicality)

$$\frac{\bullet(A \vee B)}{\bullet A \vee \neg A} \qquad \frac{\bullet(A \vee B)}{\bullet B \vee \neg B} \qquad \frac{\bullet(A \vee B)}{(\bullet A \vee A) \vee (\bullet B \vee B)}$$

$$\frac{\bullet(A \wedge B)}{\bullet A \vee A} \qquad \frac{\bullet(A \wedge B)}{\bullet B \vee B} \qquad \frac{\bullet(A \wedge B)}{(\bullet A \vee \neg A) \vee (\bullet B \vee \neg B)}$$

$$\frac{\bullet A \wedge \bullet B}{\bullet(A \vee B)} \qquad \frac{\bullet A \wedge \bullet B}{\bullet(A \wedge B)}$$

$\bullet A \vee A$ means ‘the falsity of $A$ is excluded’

$\bullet A \vee \neg A$ means ‘the truth of $A$ is excluded’

# References


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**Muito obrigado!**

**abilio.rodrigues@gmail.com**