Paraconsistency, information, and evidence

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Overview

Paraconsistency as preservation of evidence

- Conflicting evidence
- Information versus evidence
- 2 Logics of evidence and truth
 - The Basic Logic of Evidence BLE
 - The Logic of Evidence and Truth LET_J

Semantics

- Non-deterministic valuation semantics
- Probabilistic semantics
- Inferential semantics

On paraconsistency

What does it mean to accept a contradiction?

Paraconsistent logics

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- A paraconsistent logic can accept contradictions without triviality.

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What is the nature of contradictions that are accepted in paraconsistent logics?

Dialetheism: true contradictions

A dialetheia is a sentence, A, such that both it and its negation, $\neg A$, are true (...) Dialetheism is the view that there are dialetheias. (...) dialetheism amounts to the claim that there are true contradictions. (Priest and Berto, Dialetheism, Stanford.)

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Such a contradictory object really exists??!! I don't think so.

Contradictions as conflicting evidence

In order to:

- 1. reject dialetheism,
- 2. reject a metaphysically neutral position about contradictions, and
- 3. endorse a paraconsistent logic,

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- A property weaker that truth: a proposition A may enjoy such a property even if A is not true.
- 'Evidence that A is true' \rightsquigarrow 'reasons for believing in A',
- 'Evidence that A is false' \rightsquigarrow 'reasons for believing in $\neg A$ '.
- Non-conclusive evidence can be contradictory and incomplete.

Jon Michael Dunn on information

I like to think of information, at least as a first approximation, as what is left from knowledge when you subtract, justification, truth, belief, and any other ingredients such as reliability that relate to justification. Information is, as it were, a mere "idle thought." Oh, one other thing, I want to subtract the thinker.

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[Information] is something like a Fregean "thought," i.e., the "content" of a belief that is equally shared by a doubt, a concern, a wish, etc.

(J. M. Dunn, Information in computer science, 2008, p. 589.)

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- Non-conclusive justification is a justification that might be wrong, that may end up not being a justification at all.
- Information is more general than evidence: evidence, even conclusive, is still information.

- 1. No evidence at all: both A and $\neg A$ do not hold;
- 2. Only evidence that A is true: A holds, $\neg A$ does not hold;
- 3. Only evidence that A is false: A does not hold, $\neg A$ holds;
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- A logic of evidence does not preserve truth, but rather evidence from premises to conclusion.
- Positive and negative evidence are two primitive, independent and non-complementary notions: absence of positive evidence ≠ existence of negative evidence, and so on.
- A logic of evidence has different rules for positive and negative evidence.

The Basic Logic of Evidence – BLE (N4)

Paraconsistency, information, and evidence

Extending *BLE* to a logic of evidence and truth

The logic of evidence and truth – LET_J

The Logic of Evidence and Truth (LET_J) is obtained by extending the language of *BLE* with a classicality operator \circ and adding the following inference rules:

$$\frac{\circ A \quad A \quad \neg A}{B} \quad EXP^{\circ} \qquad \frac{\circ A}{A \lor \neg A} \quad PEM^{\circ}$$

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- The operator \circ works as a **context switch**: if $\circ A, \circ B, \circ C...$ hold, the *argumentative context* of A, B, C... is classical.
- A proposition $\circ A$ may be read as:
 - 1. A behaves classically,
 - 2. conclusive evidence for A or $\neg A$,
 - 3. reliable information about A or $\neg A$,
 - 4. the truth-value of A has been established.

The intended interpretation of LET_J

- When $\circ A$ does not hold, four *non-conclusive* scenarios:
 - 1. A holds, $\neg A$ doesn't \rightsquigarrow only evidence that A is true.
 - 2. $\neg A$ holds, A doesn't \rightsquigarrow only evidence that A is false.
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- When $\circ A$ holds, two *conclusive* scenarios:
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Semantics

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- Non-deterministic valuation semantics: 'mathematical representations' of the deductive systems, rather than explanations of meanings.
- **Probabilistic semantics**: intends to quantify the evidence attributed to a proposition *A* (joint work with J. Bueno-Soler and W. Carnielli).
- Inferential semantics: meanings are explained compositionally by means of the introductions rules, analogously to the proof-theoretic semantics for intuitionistic logic.

Non-deterministic valuation semantics

- Given a language *L*, valuations are functions from the set of formulas of *L* to {0,1} according to certain conditions that somehow 'represent' the axioms and/or rules of inference.
- The attribution of the value 0 to a formula A means that A does not hold, and the value 1 means that A holds.

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- The attribution of the value 0 to a formula A means that A does not hold, and the value 1 means that A holds.
- The valuation semantics for *BLE* and *LET*_J:
- 1. Provide decision procedures by means of the so-called quasi-matrices
- 2. Are non-deterministic the semantic value of $\neg A$ is not functionally determined by the semantic value of A.

Carnielli and Rodrigues. An epistemic approach to paraconsistency: a logic of evidence and truth. In *Synthese*, 2017. Preprint: http://bit.ly/SYNLETJ.

Valuation semantics for BLE and LET_J

p
ightarrow (
eg p
ightarrow q)

р	0					1						
$\neg p$	0			1			0			1		
q	0		1	C	0		()	1	0	1	
eg p ightarrow q	()	1	1	()	1	0	1	1	0	1
p ightarrow (eg p ightarrow q)	0	1	1	1	0	1	1	0	1	1	0	1
	<i>s</i> ₁	<i>s</i> ₂	<i>s</i> 3	<i>s</i> 4	<i>S</i> 5	<i>s</i> ₆	<i>S</i> 7	<i>s</i> ₈	S 9	<i>s</i> ₁₀	<i>s</i> ₁₁	<i>s</i> ₁₂

Decision procedure for BLE and LET_J

 $p \rightarrow (\neg p \rightarrow q)$ is invalid in *BLE*.

р	0					1						
$\neg p$	0			1		0			1			
q	0		1	0		1	()	1	0	1	
eg p ightarrow q	()	1	1	()	1	0	1	1	0	1
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Given s_{11} , s_1 is a valuation.

Decision procedure for BLE and LET_J

 $\circ p \rightarrow (p \lor \neg p)$ is valid in *LET*_J.

р		С)		1		
$\neg p$	0		1		0		1
• <i>p</i>	0		0	1	0	1	0
$p \lor \neg p$	0		1	1	1	1	1
$\circ ho ightarrow (ho ee \neg ho)$	0 1		1	1	1	1	1
	<i>s</i> ₁	s ₂	<i>s</i> 3	<i>s</i> 4	<i>S</i> 5	<i>s</i> ₆	S 7

 s_1 is not a valuation because there is no s' such that $s'(\circ p)=1$ and $s'(p\vee \neg p)=0$

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- In the classical approach to probabilities, $P(A) + P(\neg A) = 1$.
- Incomplete scenarios: little or no evidence for and against A. $P(A) + P(\neg A) < 1$
- Contradictory scenarios: conflicting evidence for A.
 P(A) + P(¬A) > 1

The classicality operator \circ

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- P(A) may be read as a measure of the risk associated to A, and P(○A) may be the risk of the risk associated to A.
- P(A) may even express the degree of belief in A, and P(○A) the degree of reliability of this belief.

From LET_J **to** LET_F

Problems:

- i. There is no plausible interpretation for the half-intuitionistic implication of *LET_J* in probabilistic terms.
- ii. The absence of theorems of the form $A_1 \vee \cdots \vee A_n$ that could be used to prove total probability theorems.

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Solutions:

- i. We dropped the implication of BLE/N4. The result is the well-known Belnap-Dunn's logic of first-degree entailment (FDE).
- ii. We added a non-classicality operator \bullet dual to the classicality operator \circ , and $\circ A \lor \bullet A$ is a theorem.

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The logic so obtained we called the logic of evidence and truth based on $FDE - LET_F$.

The logic of first-degree entailment (FDE)

$$\frac{A}{A \wedge B} \wedge I \qquad \frac{A \wedge B}{A} \wedge E \qquad \frac{A \wedge B}{B}$$

$$\frac{A}{A \wedge B} \wedge I \qquad \frac{B}{A \vee B} \qquad \begin{bmatrix} [A] & [B] \\ \vdots & \vdots \\ \vdots & \vdots \\ A \vee B & C & C \\ C & \forall E \end{bmatrix}$$

$$\frac{A}{\neg (A \wedge B)} \neg (I \qquad \frac{\neg B}{\neg (A \wedge B)}) \qquad \frac{\neg (A \wedge B)}{C} \qquad \frac{C}{C} \qquad \nabla E$$

$$\frac{\neg A}{\neg (A \vee B)} \neg (I \qquad \frac{\neg (A \vee B)}{\neg A} \rightarrow \forall E \qquad \frac{\neg (A \vee B)}{\neg B}$$

$$\frac{A}{\neg \neg A} DN \qquad \frac{\neg \neg A}{A}$$

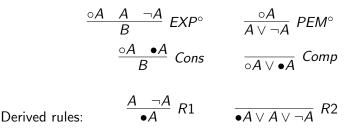
 $LET_F = FDE + \text{the following rules for } \circ \text{ and } \bullet$:

$$\frac{\circ A \quad A \quad \neg A}{B} \quad EXP^{\circ} \qquad \frac{\circ A}{A \lor \neg A} \quad PEM^{\circ}$$
$$\frac{\circ A \quad \bullet A}{B} \quad Cons \qquad \frac{\circ A}{\circ A \lor \bullet A} \quad Comp$$

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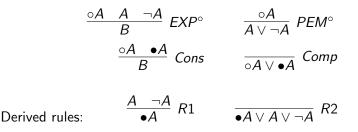
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Derived rules:
$$\frac{A \quad \neg A}{\bullet A} R1 \qquad \frac{\bullet A \lor A \lor \neg A}{\bullet A \lor A \lor \neg A} R2$$

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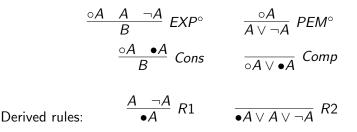
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Either *there is* conclusive evidence, or *there is not* conclusive evidence. •*A* implies conclusive evidence (or reliable information). Non-conclusive evidence (or unreliable information) implies •*A*.

Non-deterministic valuation semantics for LET_F

The quasi-matrix below displays the behavior of \circ and \bullet in LET_F .

A		0		1			
$\neg A$	0	1	L	0		1	
οA	0	1 0		1	0	0	
●A	1	0 1		0	1	1	

- Conflicting evidence implies $v(\bullet A) = 1$ and $v(\circ A) = 0$.
- No evidence at all implies $v(\bullet A) = 1$ and $v(\circ A) = 0$.
- If exactly one holds between A and ¬A, then v(●A) and v(○A) are undetermined.

In order to say that A is true, or false, evidence for the truth, or for the falsity, of A is not enough. We need conclusive evidence.

The intended interpretation of LET_F

- When *A* holds, four *non-conclusive* scenarios:
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Back to probabilistic semantics

Probability distributions

Given a logic \mathcal{L} , with a derivability relation \vdash and a language L, a probability distribution for \mathcal{L} is a real-valued function $P : L \mapsto \mathbb{R}$ satisfying the following conditions:

- 1. Non-negativity: $0 \le P(A) \le 1$ for all $A \in L$;
- 2. Tautologicity: If $\vdash A$, then P(A) = 1;
- 3. Anti-Tautologicity: If $A \vdash$, then P(A) = 0;
- 4. Comparison: If $A \vdash B$, then $P(A) \leq P(B)$;
- 5. Finite additivity: $P(A \lor B) = P(A) + P(B) P(A \land B)$.

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 - These clauses define probability functions for both FDE and LET_F

Completeness of the probabilistic semantics

Definition

A probabilistic semantic relation \Vdash_P for LET_F is defined as: $\Gamma \Vdash_P A$ if and only if for every probability function P, if $P(B) \ge \lambda$ for every $B \in \Gamma$, then $P(A) \ge \lambda$.

Theorem

Completeness of LET_F with respect to probabilistic semantics: $\Gamma \vdash A$ if and only if $\Gamma \Vdash_P A$

Bueno-Soler, J. W. Carnielli, and A. Rodrigues. Measuring evidence: a probabilistic approach to an extension of Belnap-Dunns Logic. Manuscript in preparation.

What is next?

- The probabilistic semantics is not really talking about events, but rather about the information related to such events, constituted by propositions $A, \neg A, \bullet A, \circ A$, and other propositions formed with them.
- These propositions represent evidence that can be non-conclusive, incomplete, contradictory, more reliable or less reliable, and sometimes conclusive.

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- An information space is divided by *LET_F* in parts that are exhaustive but may be non-exclusive.
- We cannot rely on the classical, mutually exclusive partitions of the sample space.

Propagation of classicality

Propagation rules:	$\frac{\circ A}{\circ \circ A} \qquad \frac{\circ \circ A}{\circ A}$	$\frac{\circ A}{\circ \neg A}$	$\frac{\circ \neg A}{\circ A}$
Introduction rules \wedge :	$rac{\circ A \neg A}{\circ (A \wedge B)}$	$rac{\circ B \neg B}{\circ (A \wedge B)}$	$rac{\circ A \circ B}{\circ (A \wedge B)}$
Introduction rules \lor :	$\frac{\circ A A}{\circ (A \lor B)}$	$\frac{\circ B B}{\circ (A \lor B)}$	$\frac{\circ A \circ B}{\circ (A \lor B)}$
Elimination rules:	$\frac{\circ (A \land B)}{\circ A \lor \circ B}$	$\frac{\circ (A \lor B)}{\circ A \lor \circ B}$	

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Introduction of classicality

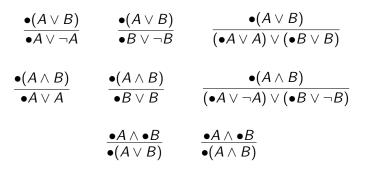
$$\frac{\circ A}{\circ (A \lor B)} \qquad \frac{\circ B}{\circ (A \lor B)}$$

If A is true, $(A \lor B)$ is true, and so is classical (m.m. for B). If A is true, it cannot be that $A \lor B$ is false – it would imply $\neg A$ and triviality.

$$\frac{\circ A \quad \circ B \quad \neg A \quad \neg B}{\circ (A \lor B)}$$

If both A and B are false, $(A \lor B)$ is false, and so classical (m.m. for B).

Derivable rules for • (non-classicality)



• $A \lor A$ means 'the falsity of A is excluded' • $A \lor \neg A$ means 'the truth of A is excluded'

References

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- W. Carnielli, and A. Rodrigues. An epistemic approach to paraconsistency: a logic of evidence and truth. Synthese. DOI: 10.1007/s11229-017-1621-7, 2017. Preprint available at http://bit.ly/SYNLETJ.
- Bueno-Soler, J. W. Carnielli, and A. Rodrigues. Measuring evidence: a probabilistic approach to an extension of Belnap-Dunns Logic. Manuscript in preparation.

Muito obrigado!

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